Probabilistic Operation of Power Systems with a Large Number of Wind Farms Considering Calculation Accuracy

Akira Koide, Tran Nguyen, and Takao Tsuji
Yokohama National University, Japan

Abstract

It is of prime importance to maintain supply and demand balance in power systems to realize the stable power supply without outage. At this time, system operators have to determine generation dispatch of generating plants considering the power flow constraint on transmission lines. However, in power systems with a large number of renewable energy, it becomes a difficult issue to keep the supply and demand balance under the large and uncertain fluctuation of those output. Though the authors have developed probabilistic system operation methods by evaluating the proper stability margin based on the probabilistic density curve, it is needed to verify its calculation accuracy. Hence, in this paper, the accuracy of the proposed method is discussed especially considering the correlated probabilistic fluctuation model and the impact of the upper and lower limit of generation output.

Keywords: power system, wind turbine, economic load dispatch, load frequency control, probabilistic operation, monte-carlo simulation

1. Introduction

To solve the energy and environmental issues, it is promoted to introduce a large number of wind turbines into power systems. Since the output of the wind turbine (WT) is weather dependent, its fluctuation gives a great impact on supply and demand balance in power systems. System operators have to determine the generation dispatch among controllable generators in order to realize the stable power supply under the uncertain fluctuation of the WT output. Therefore, the authors have developed so far new probabilistic operation methods which can maintain the power flow fluctuation within the thermal limit of transmission lines. Specifically, the fluctuation of WT output and the power flow are represented as probabilistic
density functions based on monte-carlo simulation. By using these functions, the generation dispatch is modified to minimize the fuel cost with satisfying constraint conditions from a probabilistic viewpoint.

However, the accuracy of the analysis might not be sufficient because we had some non-realistic assumptions to simplify the problem, such as the behavior of WT or the impact of the upper and lower limits of generators. Hence, in this paper, we analyze the accuracy of the simulation considering above problems to show the effectiveness of the proposed method. The rest of this paper is organized as follows. In section 2, the outline of the proposed operation method is described and its accuracy is discussed. The accuracy of the proposed method is shown using power system model in New Zealand in section 3. Finally, conclusions will be provided in section 4.

2. Probabilistic Operation considering Power Flow Constraint

2.1 Economic Load Dispatch

To realize the stable power supply, the power flow on all transmission lines must be maintained within the “thermal limit”. System operators usually determine the generation dispatch among synchronous generators included in the system considering this constraint. Since the consideration of the economic efficiency is crucial, the purpose of this operation is to minimize the total fuel cost and it is defined as “Economic Load Dispatch (ELD)”. Conventionally, ELD works to compensate the long-term demand change whose cycle is longer than roughly 30 minutes. However, in the case of power systems with many renewable energy, the apparent demand change includes their large fluctuation. Therefore, system operators have to consider the severe demand change whose magnitude is larger and cycle is shorter than conventional one. In our proposed method, to overcome this difficulty caused by the uncertain renewable energy output, the stability margin in ELD is adjusted carefully in order to avoid the excessive reduction of the economic efficiency with satisfying the thermal limit from a probabilistic viewpoint. The ELD can be formulated as follows. Where, only the WT is treated as the renewable energy.

\[
C_{ELD} = \sum_{i}^{N_G} f_i(P_{Gi}) \rightarrow \min. \quad (1)
\]

\[
f_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (2)
\]

\[
\sum_{i}^{N_G} D_i = \sum_{i}^{N_G} P_{Gi} + \sum_{i}^{W} P_{WTi} \quad (3)
\]

\[
P_{Gi} - P_{Li} - V \sum_{k=1}^{N} V_k \{G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)\} = 0 \quad (4)
\]
\[ Q_{Gi} - Q_{Li} - V_i \sum_{k=1}^{N} V_k \{ G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k) \} = 0 \quad (5) \]

\[ P_{Gi,\text{min}} \leq P_{Gi} \leq P_{Gi,\text{max}} \quad (6) \]

\[ | P_{ij} | \leq P_{ij,\text{max}} \quad (7) \]

where,

\( C_{ELD} \): total generation cost
\( f_i \): cost function of generator \( i \)
\( P_i \): active power generation of generator \( i \)
\( G \): number of generators
\( a_i, b_i, c_i \): cost coefficients of generator \( i \)
\( D_i \): demand of bus \( i \)
\( P_{WFi} \): expected output of WF \( i \) calculated from PDFs of WFs
\( Nd \): number of loads
\( Ng \): number of generators
\( W \): number of WFs
\( P_{G,i}, Q_{G,i} \): total amount of active and reactive power output at node \( i \)
\( P_{L,i}, Q_{L,i} \): active and reactive power load at bus \( i \)
\( V_i, \theta_i \): voltage and phase angle at bus \( i \)
\( V_k, \theta_k \): voltage and phase angle at bus \( k \)
\( G_{ik}, B_{ik} \): conductance and susceptance of between node \( i \) and \( k \)
\( N \): number of buses in system
\( P_{i,\text{min}}, P_{i,\text{max}} \): lower and upper limits of generation of generator \( i \)
\( P_{i,l} \): power flow on transmission line \( i \)
\( P_{i,l,\text{max}} \): upper limit of power flow on transmission line \( i \)

This optimization problem is solved by PSO algorithm in this paper.

### 2.2 Probabilistic ELD Method for Risk Control

Though ELD is the supply and demand balance control whose control cycle is longer than 30 minutes, the fluctuation cycle of the WT output should be shorter than this cycle. To compensate this faster fluctuation, the Load Frequency Control (LFC) is utilized to maintain the supply and demand balance in short time. In this control method, generation output are automatically adjusted based on the frequency and tie-line power flow information without considering the economy of the power supply. In this paper, LFC is formulated as follows supposing that the all the generators compensate the supply and demand unbalance in proportional to their rated capacities.
\[ \Delta P_{G,j} = UB \frac{P^*_{G,j}}{\sum_i P^*_{G,j}} \]  

(8)

where,

- \( UB \): the amount of the unbalance due to output fluctuation of WFs
- \( P^*_{G,i} \): rated output of generator \( i \)

WT output needs to be treated as the expected value in ELD algorithm because the control cycle of ELD is longer than the WT output fluctuation. Therefore, the power flow constraint is also satisfied considering only the expected value of the power flow. However, the power flow on all the transmission lines might change due to the LFC control. Even though the expected power flow is within the allowable range, it has a possibility that the power flow becomes beyond the upper or lower limit due to the power flow fluctuation caused by the WT output fluctuation and LFC.

Because the output fluctuation of WTs has a strong uncertainty, its impact on the power flow profile has to be evaluated by stochastic approaches. Therefore, in our proposed method, power flow changes on all transmission lines are expressed as probabilistic density curves.

If we can get the probabilistic density curves of the power flow on transmission lines, the generation dispatch is adjusted in order that all the density curves are controlled within the thermal limit. From a probabilistic viewpoint, however, the constraint violation with an extremely low probability should be allowed in the practical way. Therefore, we defined “Available Transmission Capacity” and “Security Index” which are the virtual thermal limit and maximum allowed probability of the constraint violation, respectively. The Security Index can be determined by system operators arbitrary considering the tradeoff relationship between the security of the system operation and its economic efficiency. In order to avoid that the overloading probability is larger than the Security Index, the thermal limit on all the transmission lines are virtually adjusted based on the cumulative function of the power flow fluctuation. This Available Transfer Capacity is utilized in ELD instead of actual thermal limit. As shown in fig.1, the expected value of the power flow is lowered by introducing Available Transfer Capacity and it is realized to control the violation probability supposing that the shape of the density curve does not change at this time.
Fig. 1  Risk control by adjusting the expected value of the power flow fluctuation.

2.3  Monte-Carlo Simulation for Probabilistic Density Curves

Generally, the wind velocities in multiple areas have correlation each other, it is also important to consider the correlation for determining WT output in probabilistic simulation. In this paper, the probabilistic density curve of power flow is derived by monte-carlo simulation because it is more difficult to consider the correlation in the analytical method. Thus, it is described how to generate the correlated samples to model the WT output in this section.

First, an arbitrary continuous random number $X$ shown by a probability distribution function can be transformed into uniform section $[0,1]$ by the application of cumulative distribution function (CDF). On the contrary, the invertible CDF can transform a uniform variable to its actual domain. The uniform variable is given in interval $[0,1]$.

By this definition, for a stochastic variable $X$ with the CDF given as $F_X(X) = P(X \leq x)$, the value of $F_X(X)$ is distributed on interval $[0,1]$. This relationship becomes a tool for sampling in MCS.

$$U = F_X(X) \leftrightarrow X = F_X^{-1}(U) \quad (X)$$
To generate the samples of probabilistic variables $X$ with a CDF $F_X(X)$, first random realizations $u$ of a uniform variable $U$ on the interval $[0,1]$ are generated. The value of $u$ equals the accumulative probability of $X$. Using the random realization $u$, the samples of the actual variable are obtained by the equation $X = F_X^{-1}(u)$ in MCS. The procedure is shown in Fig.2.

![Fig.2 Sampling of an arbitrary random variable](image)

### 2.4 Accuracy of the Proposed Method

The accuracy of the proposed method should be verified from the following two viewpoints.

(i) Though the Joint Normal Transform method is utilized to generate the correlated samples in monte-carlo simulation, its accuracy depends on the degree of its nonlinearity. In the case of the nonlinear function, it is expected that the correlation coefficient calculated by generated samples is different from the actual value while there should be very small error in the case of linear function. Hence, the validity of this method must be verified.

In addition, there is a possibility that we can improve the accuracy by applying the ranking method even in the case of nonlinear functions. In this method, $X$ and $Y$ values are transformed into the ranking without using their raw values because the ranking relationship can be preserved also in the case of nonlinear functions.

(ii) As described in the previous section, it is supposed that the shape of the density function does not change in the case that its expected value is adjusted virtually. Actually, however, there is a possibility that the shape of the density curves change if the generation output at any generator reaches its maximum value because it cannot participate in LFC control. This impact will be also verified next section.
3. Simulation Results

3.1 Simulation Model

The reduced power system model in New Zealand is used in this section. The network structure, the upper and lower limits of generators, and the cost coefficients of generators are shown in fig.3, table 1 and table 2, respectively (see [15] for other details). The total amount of loads is 2010MW. Two WFs whose capacities are 200 MW with correlation $\rho=0.6$ are connected to bus 5 and 12. The total rated output of those WFs accounts about 25% of the system demand. Since the average wind speeds are 6m/s and 7m/s, the probability distribution of wind power of WF1 and WF2 is expressed like Fig.4. Moreover, these two generation fluctuate with the high correlation. The scatter graph is shown in fig.5 and we can see from this figure that they have strong correlation. Finally, the demand is set as maximum supposing that only the peak period is treated.
Table 1 The upper and lower limits of the generators

<table>
<thead>
<tr>
<th>Generator</th>
<th>G0</th>
<th>G4</th>
<th>G7</th>
<th>G9</th>
<th>G10</th>
<th>G14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper limit [p.u]</td>
<td>8.0</td>
<td>6.3</td>
<td>4.0</td>
<td>3.0</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Lower limit [p.u]</td>
<td>0.5</td>
<td>2.0</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2 The cost coefficients of the generators

<table>
<thead>
<tr>
<th>Generator</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>G0</td>
<td>0.05247</td>
<td>38.539</td>
<td>756.798</td>
</tr>
<tr>
<td>G4</td>
<td>0.10587</td>
<td>46.159</td>
<td>451.325</td>
</tr>
<tr>
<td>G7</td>
<td>0.02803</td>
<td>40.396</td>
<td>1049.997</td>
</tr>
<tr>
<td>G9</td>
<td>0.03546</td>
<td>38.305</td>
<td>1243.531</td>
</tr>
<tr>
<td>G10</td>
<td>0.02111</td>
<td>36.327</td>
<td>1658.559</td>
</tr>
<tr>
<td>G14</td>
<td>0.01799</td>
<td>38.270</td>
<td>1356.659</td>
</tr>
</tbody>
</table>

Fig. 4 Probabilistic density curve of wind speed.

Fig. 5 Correlation of wind speed between WT1 and WT2.
3.2 Basic Result

The proposed method is tested using the simulation model shown in 3.1. Table 3 shows the generation dispatch in original state and after the proposed method is applied. In original dispatch, the power flow on line 8-11 fluctuate fully over the upper limit. However, the power flow fluctuation is controlled within the thermal limit in the case of the proposed method as shown in Fig.6. The power flow constraint is satisfied as to the other lines.

![Fig.6 Power flow on line 8-11 in time domain.](image)

Table 3 Generation dispatch of generators.

<table>
<thead>
<tr>
<th></th>
<th>G0</th>
<th>G4</th>
<th>G7</th>
<th>G9</th>
<th>G10</th>
<th>G14</th>
</tr>
</thead>
<tbody>
<tr>
<td>original dispatch</td>
<td>151</td>
<td>100</td>
<td>400</td>
<td>227</td>
<td>428</td>
<td>400</td>
</tr>
<tr>
<td>proposed method</td>
<td>360</td>
<td>180</td>
<td>235</td>
<td>168</td>
<td>500</td>
<td>262</td>
</tr>
</tbody>
</table>

3.3 Correlated WT output

Table 4 shows the evaluation of the accuracy regarding three cases where the correlation coefficients are specified as 0.390, 0.593 and 0.796. The “accuracy” in this figure represents the percentage of the correlation coefficient generated by the proposed method to the specified value. As shown in this table, the generated correlations are a little smaller than the specified values. By applying the ranking method, the accuracy can be slightly improved. However, the impact by the nonlinearity can be negligible since the simulation results are almost the same even if the ranking method is applied in this simulation model.
Table 4  Accuracy improvement of correlation coefficient.

<table>
<thead>
<tr>
<th>specified correlation</th>
<th>accuracy with ranking method [%]</th>
<th>accuracy with original method [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.390</td>
<td>98.3</td>
<td>97.6</td>
</tr>
<tr>
<td>0.593</td>
<td>99.5</td>
<td>98.8</td>
</tr>
<tr>
<td>0.796</td>
<td>99.8</td>
<td>99.5</td>
</tr>
</tbody>
</table>

3.4 Upper and Lower Limit of Generation Output

As described in section 2, it has a possibility that the shape of the probabilistic density curve changes if any generation output reaches its maximum value in LFC control. Table 5 shows the standard deviation in original state and after the proposed method is applied. Where, the standard deviation derived by the proposed method is divided by the original value and it is treated as the index in percentage. The maximum difference is around 11[%] on transmission line 8-11. The specific probabilistic density curve of power flow on transmission line 8-11 is shown in Fig.7. Though the expected value is largely different, the shape of the curve is almost the same. We can see from this figure that the impact of the upper and lower limit of generators is not so severe in this simulation case.

In this case, the standard deviation becomes smaller in many transmission lines by the impact of the upper and lower limit while it becomes larger in some lines. From a viewpoint of the risk management, the increase of the standard deviation has to be treated carefully because it might cause the increase of the overloading probability. Table 5 shows the maximum increase of the standard deviation is around 10[%] on line 2-16.

Table 5  Ratio of standard deviation after recalculation to original value.

<table>
<thead>
<tr>
<th>line</th>
<th>index [%]</th>
<th>line</th>
<th>index [%]</th>
<th>line</th>
<th>index [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>88.98</td>
<td>3-5</td>
<td>99.09</td>
<td>11-12</td>
<td>98.82</td>
</tr>
<tr>
<td>1-2</td>
<td>99.98</td>
<td>6-7</td>
<td>99.59</td>
<td>12-13</td>
<td>99.41</td>
</tr>
<tr>
<td>1-3</td>
<td>98.70</td>
<td>6-8</td>
<td>92.24</td>
<td>12-15</td>
<td>101.0</td>
</tr>
<tr>
<td>1-5</td>
<td>99.76</td>
<td>6-16</td>
<td>99.88</td>
<td>12-16</td>
<td>99.39</td>
</tr>
<tr>
<td>2-11</td>
<td>108.4</td>
<td>8-9</td>
<td>99.32</td>
<td>13-16</td>
<td>99.40</td>
</tr>
<tr>
<td>2-16</td>
<td>110.2</td>
<td>8-11</td>
<td>98.14</td>
<td>14-15</td>
<td>98.87</td>
</tr>
<tr>
<td>3-4</td>
<td>98.87</td>
<td>10-16</td>
<td>98.89</td>
<td>15-16</td>
<td>102.6</td>
</tr>
</tbody>
</table>
4. Conclusions

It is an important issue to develop new ELD technologies which can treat the uncertain fluctuation of renewable energy. The authors have so far developed a probabilistic ELD approach for power systems with a large number of WTs. However, some discussion have been needed in the accuracy of the proposed method especially related to the accuracy of the probabilistic model for the correlated WT output and the upper limit of generation output. As a result, it was shown that the accuracy of the proposed method is not so problematic though it includes some errors. The future works are as follows:
There are some other constraints to consider in ELD such as voltage stability, frequency control, and synchronous stability. The proposed method will be expanded in order that it can treat those constraints.

The effectiveness and the accuracy of the proposed method must be clarified using very large scale power system models based on the actual power systems.

References